

2023-24 CMSE114312023 AND PRESCRIPTIVE ANALYTICS WITH MATHEMATICAL PROGRAMMING

Optimising Crop Allocation in India with Mathematical Approach – Mixed Integer Programming

B236310, B244724, B236816, B235064, B244512

Word count: 4014

**1. Introduction**

Agriculture holds a paramount position in the global economy, especially in underdeveloped countries. It serves as a vital source of labor, income, and sustenance. While the role of agriculture in international economic development has improved significantly over the past few years, there are still many countries where agriculture is not as effective as it could be. According to the World Bank, the growth of the agricultural sector is two to four times more effective in elevating the income of the impoverished compared to other industries.

India, often dubbed an agricultural country, heavily relies on its agriculture sector. As highlighted by Grewal & Daneshyari (2022), 58 % of its citizens rely on farming as a livelihood source, making its country one of the top global agricultural houses producing farming products. Patil et al. (2020) further underscore the significance of the agriculture sector by revealing that it employs roughly half of India’s labor force. Despite these substantial contributions to various aspects of India, the agriculture sector presents a paradoxical situation with respect to its contribution to the country’s GDP. Of the approximately 3 trillion USD total of GDP, the agricultural sector accounts for only around 18%. (Figure 5, World Bank, 2023).

The severe imbalance between the labor market’s dependence on agriculture and its relatively modest contribution to the Indian GDP necessitates immediate steps to enhance agricultural efficiency. Optimal crop allocation emerges as a pivotal aspect of agricultural management, potentially maximizing agricultural productivity. Efficient crop allocation can lead to several benefits, including resource optimization, increasing economic returns, and bolstering environmental sustainability.

Mathematically modelling has been applied in diverse ways to optimize crop allocation in agriculture. Jain et al. (2018) conducted a comprehensive review of various methods dedicated to crop allocation, delving into the seasonality of crop planning. Custodio et al. (2023) applied Linear Programming (LP) to optimize crop harvesting plans and land allocation, as it can deal with various variables and constraints simultaneously. While various LP models have been developed for solving crop allocation problem, each model incorporates different considerations and constraints. Jain et al. (2018) emphasized the potential biases in models that solely consider land size and farmgate prices, advocating for the inclusion of other factors like water resources, soil conditions and labor requirements. Bhatia & Rana (2020) proposed that when optimizing the crop pattern, the farmers need to understand their land resources. Because it helps to maximize the reallocation efficiency of the crop allocation. Yu et al. (2022) particularly pointed out the benefit of crop rotation, which not only improve the climate resilience of the agricultural production system, but also contribute to the crop productivity.

Building on prior research, this paper employs Mixed Integer Programming to address the crop allocation challenge in India. Additionally, this paper explores the impact of rotation benefits on total agriculture profit. By incorporating real-world data and a comprehensive array of factors, this paper is more significant in practice compared to others.

**2. Problem Description**

During the period of 2011-2015, the demand and production dynamics of crops in India underwent a complex transformation. Crops, being a pivotal element in ensuring food security, were subject to a confluence of factors, including demographic shifts, evolving dietary preferences, and governmental strategies. That is why the objective focused on utilizing optimization strategies to improve farm efficiency. Specifically, maximizing the profit of agriculture by reallocating 8 crops planting across 33 states in India. 8 crops, such as: rice, wheat, maize, cotton, jute, horse gram, soyabean, potato, are selected based on demand, statistical indicators, and research work listed above.

In summary, the 2011-2015 period in India marked a transformative phase for crop dynamics, emphasizing the commitment to optimize cereal production through initiatives like the National Food Security Mission. The focus on optimizing planting areas for 8 key crops across states aimed at enhancing farm efficiency and maximizing profits. Adhering to a 27% maximum allocation per crop within a state added a crucial dimension, considering variations in yield influenced by environmental factors. The study's nuanced approach to performance analysis integrated fertilizer indicators from leading companies, contributing to a comprehensive understanding of crop production optimization.

**2.1. Data Description**

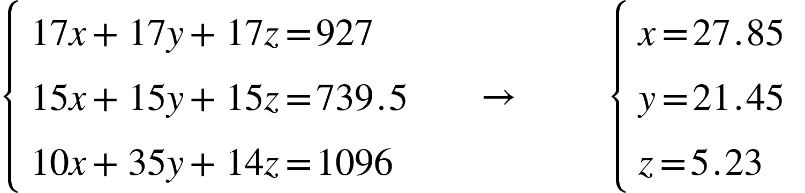
In the data description section of the research paper, we provide a comprehensive overview of the datasets used in this study.

A diagram of different types of agricultural vehicles

Description automatically generatedFigure 1. Cropping patterns in the whole agriculture year in India

According to the selected dataset, the whole agriculture year is illustrated in Figure.1. We focused on 8 dominant crops which collectively occupy 70% of the total cultivated area in India. Which brings us to the next assumption that is present in the work - ensuring that the allocation of planted area for each crop within each state does not exceed 27%. The maximum territory occupied by one crop does not exceed 27%, according to the dataset, which allows us to analyze at least 4 crops in the territory we are considering. Since our main goal is to maximize profits through performance analysis, the yield indicator is considered. And the next assumption is recognizing that variations in yield across states include the influence of factors such as temperature, rainfall, and soil pH level.

Fertilizer indicators are used in further calculations of the mathematical model. According to the Ministry of Chemicals and Fertilizers in India (2022), for the period under review in India, the leading position in the market was occupied by 3 fertilizer companies - Indian Farmers Fertiliser Cooperative Limited (IFFCO, www.iffco.in), Krishak Bharati Cooperative Limited (KRIBHCO, www.kribhco.net), and Coromandel International Limited (Coromandel International Limited, www.coromandel.biz) We have compiled a system of equations based on prices for fertilizers and the proportions of N, P and K contained in them. In this system of equations:



Where x is the amount of substance N in one kilogram of fertilizers, y is the amount of substance P in one kilogram of fertilizers, z is the amount of substance K in one kilogram of fertilizer.

These indicators of price and proportion of the substance are the average indicator among the three above-mentioned companies providing fertilizers in India for the period under consideration (from 2011 to 2015). The exact amount of fertilizer required for each variety is calculated based on the indicators provided in the dataset. The total demand for each crop variety was calculated based on the economic formula of Import and Export, according to data provided by the Government of India, Ministry of Commerce and Industry. Number of working days in each time and Daily wage for workers in each state are taken from data provided by The Ministry of Agriculture and Farmers Welfare and the Ministry of Labor & Employment, respectively. In addition, in the further mathematical equation there is a fixed cost for the rental charge of farm machinery per agriculture period, which is also taken from the statistical data of The Ministry of Agriculture and Farmers Welfare. The Minimum Support Price (MSP) is a government-initiated policy in India that establishes a baseline price for various agricultural commodities to ensure remunerative returns for farmers. Set by the government agencies, MSP aims to provide a safety net for farmers by guaranteeing a minimum price for their produce, particularly essential crops like rice, wheat, and pulses. This intervention serves to stabilize agricultural incomes, promote crop diversification, and contribute to food security by encouraging farmers to cultivate staple crops. Since the price in each state may vary, the price taken for one ton of crop type corresponds to the average The MSP for 2011-2015. (Pattanayak, 2016).

Rotation benefit refers to the positive effects observed when different crops are alternated or rotated in a particular sequence over time. This agricultural practice aims to enhance soil fertility, mitigate pest and disease pressures, and optimize resource use. Crop rotation contributes to improved yields, increased nutrient efficiency, reduced reliance on chemical inputs, and overall sustainability. Additionally, it can lead to economic advantages by diversifying farm income and minimizing risks associated with monoculture. Rotation benefits are pivotal in promoting resilient and productive agricultural systems. According to the literature analyzed by the group, the three most profitable and most important rotational pairs of crops were identified. Jat et al. (2020) demonstrated that the rotation benefit of rice and wheat results in a notable 5.8% increase in net income by 25.9%. Chi et al. (2023) concluded that the maize yield increased by approximately 5.63% in 2001 and 7.69% in 2002 when it was planted following cotton, compared to when it was planted continuously. And the rotation benefit from cotton to soybean can be counted as 23%. Additionally, Lalsanglur (2021) applied the productivity indicators of the selected crops were taken and divided by the amount of cultivated land to calculate yield.

**3. Mathematical Formulation**

Table 1. Mathematical Notation

|  |  |
| --- | --- |
| **Sets** | **Description** |
|  | 8 types of crops |
|  | 33 States of India |
|  | 3 agricultural periods of time |
| **Parameters** | **Description** |
|  | Yield of crop in state during time |
|  | Price of crop |
|  | Demand of crop |
|  | Land area of state during time |
|  | Seed cost for crop |
|  | Seeding rate for crop |
|  | Average daily wage in state |
|  | Working days during time |
|  | Number of workers needed in state |
|  | Fertilizer cost for crop |
|  | Fixed cost for machine renting in state during time |
| \* | Benefited area specifically equal to 0 when |
|  |  |
| **Decision Variables** | **Description** |
|  | Land area allocated for crop in state during time |
|  | Binary Variable indicating if any crop is planted in state during time |
| \* | Area benefited from the rotation for crop in state during time |
|  |  |
| **Scalar** | **Description** |
| LandUseRate | Land using rate for chosen crops |
| MaxCropCoverage | A maximum of one crop can be grown in each area |
| SmallNumber | /0.000001/ |
| LargeNumber | /1000000000/ |
| \*RotationBenefit\_mc | Rotation benefit between maize and cotton |
| \*RotationBenefit\_cs | Rotation benefit between cotton and soyabean |
| \*RotationBenefit\_rw | Rotation benefit between rice and wheat |

*Notes: \* specifically for Model 2*

**3.1. Decision Variables**

The first model (M1) employs several decision variables to represent the reallocation of planting areas, to maximize total profit. Specifically, the positive variable, denoted as , signifies the size of the reallocated planting area. These variables are characterized by three indexes: for the type of crop, for the state, and for the time. Consequently, a single variable represents the size of the reallocated planting area for crop in state during time . Binary variable is introduced to indicate whether any crop is planted in state during time .

Furthermore, the second model (M2) introduces decision variable to represent the size of planting area for crop in state during time that benefits from crop rotation.

**3.2. Objective Function and Constraints**

In the first model (M1), we exclude considerations related to crop rotation benefit. Mathematically, it is formulated as follows:

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Subject to: (2)

(3)

(4)

(5)

(6)

The objective function (1) is designed to maximize the total profit derived from crop cultivation, by subtracting labour costs, seed costs, fertilizer costs and fixed costs from the total revenue generated by production. Constraints (2) ensure that the production of each crop meets its corresponding demand. Constraints (3) restrict the land area allocated to the selected crops in our model. Constraints (4) limit the maximum proportion of production for each crop grown in each state. Constraints (5) and (6) serve to assign values to the binary variable . If there is no crop planted in state during time , is set to 0; otherwise, it is set to 1.

The second model (M2), which incorporates crop rotation benefits, is defined as follows:

*(Scenario 1)*  {"mathml":"<math style=\"font-family:stix;font-size:16px;\" xmlns=\"http://www.w3.org/1998/Math/MathML\"><mstyle mathsize=\"16px\"><mi>m</mi><mi>a</mi><mi>x</mi><mo>&#xA0;</mo><munder><mo>&#x2211;</mo><mrow><mi>i</mi><mo>&#x2208;</mo><mi>I</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>Y</mi><mrow><mi>i</mi><mi>j</mi><mi>t</mi></mrow></msub><msub><mi>R</mi><mrow><mi>i</mi><mi>j</mi><mi>t</mi></mrow></msub><msub><mi>P</mi><mi>i</mi></msub><mo>-</mo><munder><mo>&#x2211;</mo><mrow><mi>i</mi><mo>&#x2208;</mo><mi>I</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>R</mi><mrow><mi>i</mi><mi>j</mi><mi>t</mi></mrow></msub><mi>D</mi><msub><mi>W</mi><mi>j</mi></msub><mi>W</mi><msub><mi>D</mi><mi>t</mi></msub><mi>N</mi><msub><mi>W</mi><mi>j</mi></msub><mspace linebreak=\"newline\"/><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>-</mo><munder><mo>&#x2211;</mo><mrow><mi>i</mi><mo>&#x2208;</mo><mi>I</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>R</mi><mrow><mi>i</mi><mi>j</mi><mi>t</mi></mrow></msub><mi>S</mi><msub><mi>C</mi><mi>i</mi></msub><mi>S</mi><msub><mi>R</mi><mi>i</mi></msub><munder><mrow><mo>-</mo><mo>&#x2211;</mo></mrow><mrow><mi>i</mi><mo>&#x2208;</mo><mi>I</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>R</mi><mrow><mi>i</mi><mi>j</mi><mi>t</mi></mrow></msub><mi>F</mi><mi>e</mi><mi>r</mi><msub><mi>C</mi><mi>i</mi></msub><mo>-</mo><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><mi>C</mi><msub><mi>P</mi><mrow><mi>j</mi><mi>t</mi></mrow></msub><mi>F</mi><mi>i</mi><mi>x</mi><mi>e</mi><mi>d</mi><msub><mi>C</mi><mrow><mi>j</mi><mi>t</mi></mrow></msub><mspace linebreak=\"newline\"/><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>+</mo><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>Y</mi><mrow><mo>'</mo><mi>c</mi><mi>o</mi><mi>t</mi><mi>t</mi><mi>o</mi><mi>n</mi><mo>'</mo><mi>j</mi><mi>t</mi></mrow></msub><mi>R</mi><msub><mi>B</mi><mrow><mo>'</mo><mi>c</mi><mi>o</mi><mi>t</mi><mi>t</mi><mi>o</mi><mi>n</mi><mo>'</mo><mi>j</mi><mi>t</mi></mrow></msub><msub><mi>P</mi><mrow><mo>'</mo><mi>c</mi><mi>o</mi><mi>t</mi><mi>t</mi><mi>o</mi><mi>n</mi><mo>'</mo></mrow></msub><mo>&#xB7;</mo><mtext>RotationBenefit_mc</mtext><mspace linebreak=\"newline\"/><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>&#xA0;</mo><mo>+</mo><munder><mo>&#x2211;</mo><mrow><mi>j</mi><mo>&#x2208;</mo><mi>J</mi></mrow></munder><munder><mo>&#x2211;</mo><mrow><mi>t</mi><mo>&#x2208;</mo><mi>T</mi></mrow></munder><msub><mi>Y</mi><mrow><mo>'</mo><mi>s</mi><mi>o</mi><mi>y</mi><mi>a</mi><mi>b</mi><mi>e</mi><mi>a</mi><mi>n</mi><mo>'</mo><mi>j</mi><mi>t</mi></mrow></msub><mi>R</mi><msub><mi>B</mi><mrow><mo>'</mo><mi>s</mi><mi>o</mi><mi>y</mi><mi>a</mi><mi>b</mi><mi>e</mi><mi>a</mi><mi>n</mi><mo>'</mo><mi>j</mi><mi>t</mi></mrow></msub><msub><mi>P</mi><mrow><mo>'</mo><mi>s</mi><mi>o</mi><mi>y</mi><mi>a</mi><mi>b</mi><mi>e</mi><mi>a</mi><mi>n</mi><mo>'</mo></mrow></msub><mo>&#xB7;</mo><mtext>RotationBenefit_cs</mtext></mstyle></math>","origin":"MathType for Microsoft Add-in"} (7)

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(10)

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(14)

(15)

(16)

(17)

(18)

(19)

Similarly, the objective functions (7) and (8) in M2 seek to maximize the total profit from crop cultivation, but they take crop rotation benefits into consideration. Particularly, to assess the impact of rotation benefits on total profit, we have set up two scenarios. In Scenario 1, as represented by objective function (7), we consider the rotation benefits of maize and cotton, along with cotton and soyabean. In scenario 2, building upon Scenario 1, we additionally incorporate the rotation benefit of rice and wheat, which is represented by objective function (8).

The constraints (9-13) mirror constraints (2-6) from M1. In addition, constraints (14-15) determine the smaller planting area between maize and cotton, represented by . Likewise, constraints (16-17) and constraints (18-19) respectively identify the areas benefiting from the rotation of cotton and soyabean, and rice and wheat.

**4. Computational Results**

This paper presents the development and analysis of two comprehensive models designed for the optimization of crop allocation across various states and planting seasons in India. The primary objective of these models is to maximize overall agricultural profit while adhering to a set of constraints, including land availability, labour costs, and the benefits derived from crop rotation. Utilizing a Mixed Integer Linear Programming (MILP) approach, these models were rigorously formulated and implemented using the General Algebraic Modelling System (GAMS). Additionally, the results and insights derived from these models have been effectively visualized using Python, providing a clear and intuitive understanding of the optimization strategies and their potential impacts.

**4.1. Key Findings from M1**

This section delineates the pivotal findings obtained from M1, which was developed to emulate the intricacies of contemporary agricultural practices. The model's core objective revolves around maximizing the efficacious use of available land and aligning closely with prevailing market demands. Notably, M1 proposes a strategic reconfiguration of crop planting patterns, an approach that is projected to significantly augment both the overall production and profitability within the agricultural sector.

The outcomes derived from M1 underscore substantial prospects for elevating agricultural productivity and profitability. These results elucidate the advantages inherent in judicious crop distribution strategies. A quintessential highlight from M1 is the total profit realization, amounting to an impressive 21.46 trillion rupees. This figure underscores the model's efficacy in optimizing crop allocation for enhanced financial returns. More details about M1 can be found in Appendix B.

A diagram of different colored circles

Description automatically generatedFigure 2. Seasonal proportion and subdivision of crop production

*Notes: Figure 2 features a 3D pie chart at the center, showcasing the proportional seasonal distribution of eight crop types: rice, wheat, maize, potato, horsegram, cotton, soybean, and jute. Encircling this pie chart are three Nightingale rose diagrams, representing the Kharif, Rabi, and Summer seasons. These diagrams detail the subdivision of crop production per season, with the arc length, angle, and sector area of each diagram proportionally reflecting the production volume of each crop.*

As illustrated in Figure 2, the Kharif and Rabi seasons emerge as the predominant agricultural periods in India, cumulatively representing approximately 84% of the nation's total annual cropping area. An intriguing aspect of M1's findings is the diverse distribution of land allocation among the eight studied crops, particularly during the Kharif season. In contrast, the Summer season exhibits a more concentrated cultivation pattern, predominantly favouring corn and rice. M1’s insights further reveal crop-specific planting preferences: cotton, soybeans, and jute are primarily sown during the Kharif season, while wheat predominantly thrives in the Rabi season. Moreover, crops like maize and rice exhibit optimal growth when cultivated in the Kharif season. Conversely, potato and horsegram are identified as more suitable for the Rabi season, indicating season-specific adaptability of these crops.

**4.2. Comparative Analysis of M2**

This analysis delves into the effects of rotation benefits on crop area reallocation in M2, employing two distinct scenarios. Scenario 1 incorporates rotations of cotton followed by soyabean, and maize followed by cotton. Scenario 2 builds upon this by adding a rice-wheat rotation. The specifics of these scenarios are detailed in Table 2.

M2’s optimization suggests a strategic reallocation of crop areas, yielding an approximate 2% profit increase over the baseline. In Scenario 1, as per Table 2, the addition of rotation benefits leads to a profit increase of 98.50 billion rupees. Scenario 2 further boosts total profit by 477.92 billion rupees, culminating in 21.93 trillion rupees. Notably, the cotton-soybean rotation in Scenario 1 primarily augments profit directly through rotation benefits, without significantly altering crop area distribution. Conversely, the maize-cotton rotation prompts a notable reallocation, especially evident in the increased Rabi season cotton area in Meghalaya (by 45,386.250 hectares).

Table 2. Outcomes of M2 across two distinct scenarios

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Reallocated Area | Profit (trillion) | Rotation Types | Rotation Benefit | Benefit Area in Rabi | Benefit Area in Summer |
| Scenario 1 | 261,958,748 | 21.56 | cotton → soyabean | 25% | 4,291,481 | 0 |
|  |  |  | maize → cotton | 12% | 2,396,310 | 0 |
| Scenario 2 | 262,021,076 | 21.93 | cotton → soyabean | 25% | 4,291,481 | 0 |
|  |  |  | maize → cotton | 12% | 2,396,310 | 0 |
|  |  |  | rice → wheat | 25.9% | 28,467,948 | 962,672 |

In Scenario 2, the added rice-wheat rotation markedly expands allocated land for both crops — rice in Kharif and wheat in Rabi. In scenarios of land saturation, the marginal profit from increased wheat cultivation can surpass that of other crops, leading to a proportional reduction in their land area during the same agricultural season to accommodate more wheat. This comparative analysis underscores the nuanced impact of rotation benefits on crop allocation strategies, highlighting potential avenues for enhanced agricultural profitability in M2. In our focused assessment, Scenario 2 of Model 2, which involves extensive area reallocation, is compared against Model 1 to underscore the optimization benefits attributable to crop rotation. Figure 3 methodically illustrates the changes in land area dedicated to crop cultivation across different states and seasons – Kharif, Rabi, and Summer – consequent to the introduction of crop rotation benefits in Scenario 2. A notable observation is the consistency in land use across states in both the Kharif and Rabi seasons, contrasted with a prevalent trend of fallow land in the Summer. The influence of crop rotation patterns is predominantly evident in the Kharif season, with adjustments in crop land allocation observed in approximately 15.6% of the states (5 states). And the Rabi season witnesses a change in crop land strategy in one state.

Figure 3. Geospatial analysis of cropping area changes in scenario 2 of M2 and M1

A map of the world

Description automatically generated

*Notes: Figure 3 utilizes a geospatial map to illustrate cropping area shifts between Scenario 2 of M2 and M1 across 33 states. States are color-coded, with varying green shades indicating the scale of available cultivation areas; those not participating in Scenario 2's production are shown in grey. Red and blue texts mark states where crop rotation benefits increased or decreased specific crop areas, respectively.*

A significant outcome identified by the model is the advantageous rotation of rice and wheat, leading to substantial increases in their planting areas – a 4,981,188 hectare rise for rice in Kharif and a 677,436-hectare augmentation for wheat in Rabi. To facilitate these expansions, Scenario 2 necessitates reductions in the planting areas of horsegram, maize, cotton, and soybean. This strategy indicates that, with the added revenue from crop rotation, the profitability of the newly prioritized crops surpasses that of the original crops. For instance, in Rabi, in states like Odisha where land is fully utilized, an increase in wheat cultivation directly corresponds to the same amount of decrease in horsegram cultivation, implying a higher profitability for wheat inclusive of rotation benefits. Similarly, in Kharif, the rise in rice cultivation leads to a varied reduction in cotton in Himachal Pradesh, where the cropping area was not fully exploited in M1.

**4.3. Sensitivity Analysis**

This analysis explores the sensitivity of the model’s profitability and crop allocation recommendations to fluctuations in crop rotation benefits. We conducted a sensitivity analysis by varying these benefits from -10% to +15%, while keeping all other parameters constant.

Figure 4. Impact of rotation benefit variations on cropping patterns

A graph of different rotation benefits

Description automatically generated

*Note:* *Figure 4 presents a detailed analysis of how alterations in rotation benefits, ranging from -10% to +15%, influence the land allocation proportions for eight crops across different agricultural seasons: Kharif, Rabi, and Summer. The figure uses a series of stacked bars, each representing one of these three seasons, to depict the distribution of land among the eight crops under varying scenarios of rotation benefits. The left axis shows land proportion per crop, while the right axis quantifies profit increases and Rabi benefit area changes.*

Figure 4 demonstrates that varying rotation benefits moderately influences total profit. Notably, within the -5% to +5% range, changes in rotation benefits appear to have minimal impact on the benefit area during the Rabi season. However, a 10% decrease in rotation benefits leads to a 45,385 hectare decrease in the Rabi benefit area, without altering the proportion of allocated area. This suggests that profits from land allocated due to crop rotation are not as substantial as from the original land allocation. Conversely, a 10% or 15% increase in rotation benefits prompts significant reallocation of crop areas. For example, with a 10% increase, there’s an expansion in the Rabi cotton planting area to capitalize on the maize-cotton rotation benefits. At a 15% increase, the Kharif cotton area is augmented, maximizing the profitability of soybean rotation in Rabi. Therefore, rotation benefit changes exceeding 10% markedly affect both profitability and crop allocation, underscoring the model’s sensitivity to these benefits.

**5. Conclusions and Recommendations**

By considering factors such as land availability, labour costs, and the benefits of crop rotation, our models provide a comprehensive framework for maximising profitability while meeting demand. After integrating the benefits of crop rotation into Model 2, we observed an increase of 470 billion in profits, rising from 21.46 trillion to 21.93 trillion. This underscores the socio-economic advantages of enhanced yields attributable to crop rotation practices. In the context of decision-making in agricultural planning, an exemplar of rotation benefits is seen in sequential seasonal planting; for instance, planting maize in one season followed by soybeans in the next can yield a 25% increase in soybean production. Therefore, by considering the synergy of different crops grown in successive seasons on the same land, within the confines of existing soil conditions and financial budget constraints, we can optimize crop yields and, consequently, maximize profitability. The flexibility of the models enables adaptation to various scenarios, rendering them invaluable for strategic decision-making in agricultural planning.

**5.1. Recommendations**

Adjust Agricultural Policies and Subsidies. Align agricultural policies and subsidy programs with the findings of the models. Introduce incentives and support mechanisms that encourage a greater number of farmers to embrace crop rotation. This can help drive sustainable development in the agricultural sector, benefitting both farmers and the environment.

Promote Interdisciplinary Collaboration. Foster collaboration among experts from diverse fields, including agricultural science, soil science, climatology, and economics. By pooling expertise and resources, interdisciplinary teams can develop more advanced agricultural models. These models will provide more accurate and comprehensive guidance for crop management strategies, accounting for the intricate interactions between crops, soil, and climate. Such collaborative efforts are crucial for addressing the complexities and challenges inherent in modern agriculture and for driving innovation in the field. By implementing these recommendations, we can further enhance agricultural productivity, sustainability, and profitability, ultimately contributing to the well-being of both farmers and the agricultural ecosystem.

**5.2. Limitation**

This case is attributed to farmers lacking necessary capabilities, as highlighted by Grewal & Daneshyari (2022). Despite the Indian government providing raw data on the official website, farmers face challenges due to inadequate access to technology, affecting cooperative decision-making for proper pricing.

A significant limitation in Indian crop production is the widespread issue of water scarcity for irrigation. The sector heavily depends on water resources, and erratic weather patterns, coupled with inefficient water management, present challenges. Insufficient and uneven rainfall distribution, worsened by climate change, often leads to drought conditions, hampering optimal crop cultivation and affecting yield and quality (Bhatia & Rana, 2020). Groundwater overexploitation further exacerbates the problem, leading to unsustainable agricultural practices. Addressing water scarcity is crucial for sustainable crop production in India.

In the pursuit of profit maximization in crop production research, a limitation arises from potential restrictions on crop diversity across states. Prioritizing high-value or high-demand crops may reduce diversity, impacting ecological resilience and increasing vulnerability to pests and diseases (Bhatia & Rana, 2020). The logistical challenge of transporting specialized crops to diverse locations incurs higher costs, posing economic constraints. Balancing profit optimization with ecological sustainability is essential for a resilient agricultural landscape.

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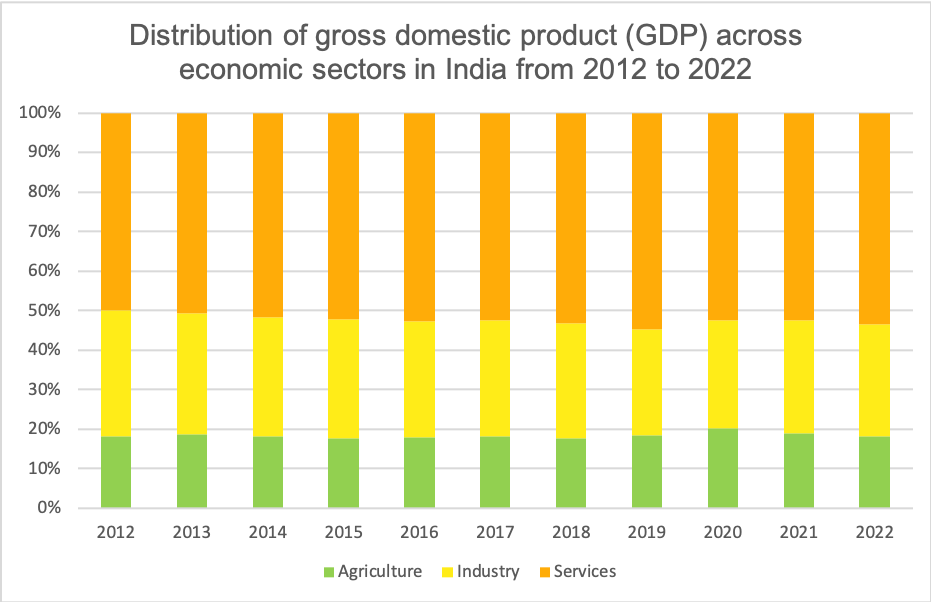
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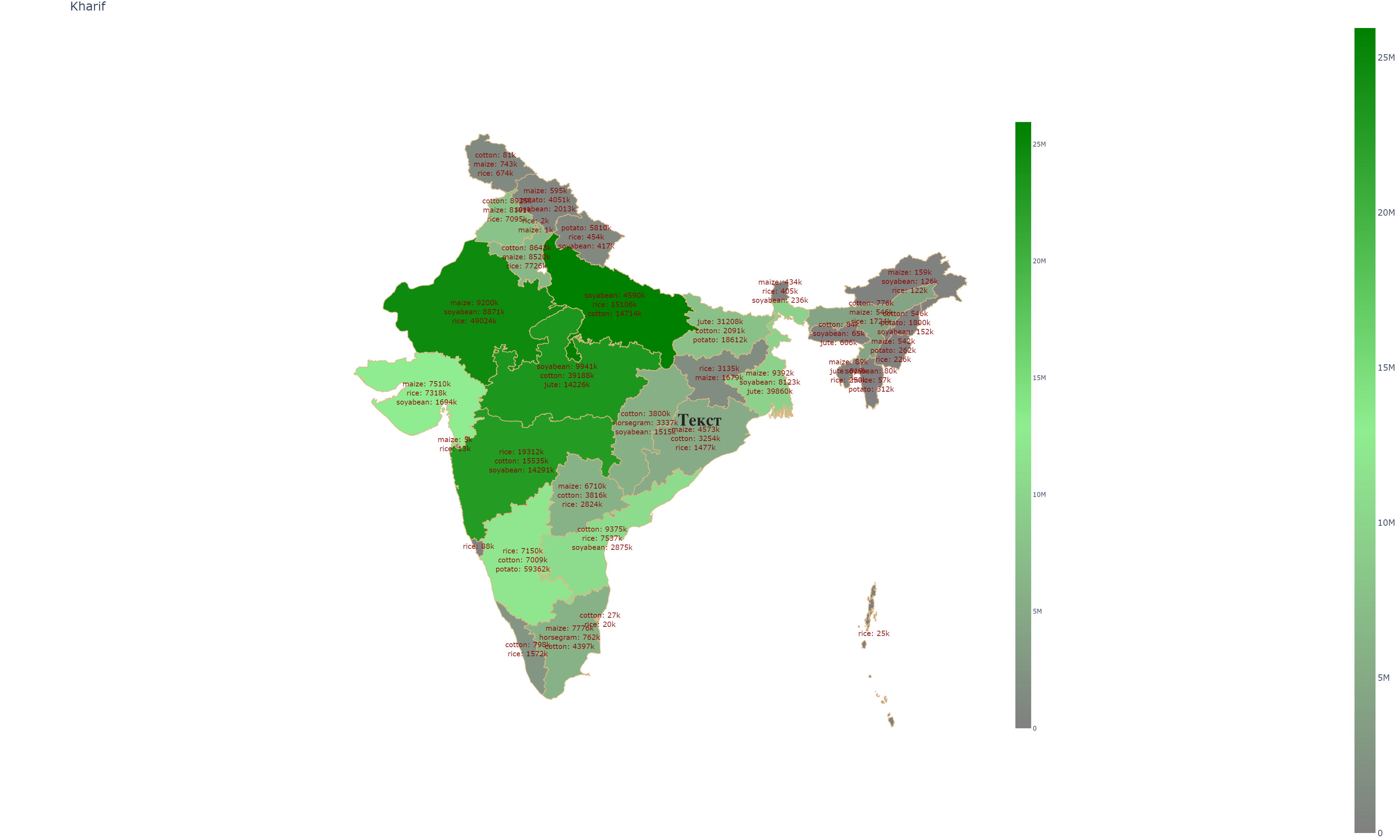
**Appendix A**

Figure 5. Distribution of GDP across economic sectors in India for the period 2012-2022.



*Notes: Figure 5 illustrates the distribution of GDP across economic sectors in India for the period 2012-2022. The data source is from World Bank, 2023.*

**Appendix B**

Figure 6. Geospatial analysis of crop production in Kharif in M1****

*Notes: Figure 6 utilizes a geospatial map to illustrate crop production in Kharif in M1 across 33 states. States are color-coded, with varying green shades indicating the scale of* *available cultivation areas; those not participating in crop production in Kharif are shown in grey. The top three crops from each state are represented in the chart.*

Figure 7. Geospatial analysis of crop production in Rabi in M1

A map of india with different shades of green

Description automatically generated

*Notes: Figure 7 utilizes a geospatial map to illustrate crop production in Rabi in M1 across 33 states. States are color-coded, with varying green shades indicating the scale of available cultivation areas; those not participating in crop production in Rabi are shown in grey. The top three crops from each state are represented in the chart.*

Figure 8. Geospatial analysis of crop production in Summer in M1

**A map of india with different shades of green

Description automatically generated**

*Notes: Figure 8 utilizes a geospatial map to illustrate crop production in Summer in M1 across 33 states. States are color-coded, with varying green shades indicating the scale of available cultivation areas; those not participating in crop production in Summer are shown in grey. The top three crops from each state are represented in the chart.*